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# Simulation model of an intelligent transportation system for the “smart city” with adaptive control of traffic lights based on fuzzy clustering

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## Abstract

This article presents a new simulation model of an intelligent transportation system (ITS) for the “smart city” with adaptive traffic light control. The proposed transportation model, implemented in the AnyLogic, allows us to study the behavior of interacting agents: vehicles (V) and pedestrians (P) within the framework

of a multi-agent ITS of the “Manhattan Lattice” type. The spatial dynamics of agents in such an ITS is described using the systems of finite-difference equations with the variable structure, considering the controlling impact of the “smart traffic lights.” Various methods of traffic light control aimed at maximizing the total traffic of the ITS output flow have been studied, in particular, by forming the required duration phases with the use of a genetic optimization algorithm, with a local (“weakly adaptive”) switching control and based on the proposed fuzzy clustering algorithm. The possibilities of optimizing the characteristics of systems for individual control of the behavior of traffic lights under various scenarios, in particular, for the ITS with spatially homogeneous and periodic characteristics, are investigated. To determine the best values of individual parameters of traffic light control systems, such as the phases’ durations, the radius of observation of traffic and pedestrian flows, threshold coefficients, the number of clusters, etc., the previously proposed parallel real-coded genetic optimization algorithm (RCGA type) is used. The proposed method of adaptive control of traffic lights based on fuzzy clustering demonstrates greater efficiency in comparison with the known methods of collective impact and local (“weakly adaptive”) control. The results of the work can be considered a component of the decision-making system in the management of urban services.

**Keywords:** intelligent transportation system, “smart city”, “smart traffic lights”, agent-based modeling, adaptive control, fuzzy clustering, AnyLogic

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## Introduction

Currently, there is an increase in the need for the design and implementation of intelligent transport systems (ITS) for a “smart city” due to the ever-increasing traffic, causing the formation of multiple traffic jams. At the same time, one of the most promising directions of the ITS evolutionary development is the use of “smart traffic lights” that analyze the dynamics and structure of traffic and pedestrian flows [1].

Various approaches to rational traffic light management are known, in particular, based on information exchange [2] using machine learning methods with reinforcement [3] based on mixed integer programming [4, 5], using genetic and swarm optimization algorithms [6–8], as well as artificial neural networks (ANS), fuzzy logic, clustering and adaptive control for the ITS [11–13].

To study the behavior and optimize the characteristics of the ITS, various combined approaches are used, for example, agent-based and discrete-event modelling methods supported in AnyLogic [14, 15], joint control of traffic lights and vehicle trajectories [16], adaptive control based on a predictive model and reinforcement learning [17]. At the same time, most of these approaches are used for ITS with a simplified configuration, for example, for two consecutive intersections [15], one intersection consisting of two roads, etc. [17]. Various scenarios that determine the periodic dynamics of interacting transport and pedestrian routes are not considered.

As a rule, significant difficulties arise when managing the characteristics of the ITS with a more complex geometry of the “Manhattan Lattice” type [18, 19]. In such an ITS, inconsistent control of the states of at least one traffic light, as a rule, leads to a change in vehicle speed and traffic density on all connected

routes. At the same time, in order to maximize the output traffic flow, it is necessary to effectively manage traffic lights, ensuring periodic prioritization between vehicles and pedestrians. So, for example, when a significant number of people gather at a regulated pedestrian crossing, the inclusion of a traffic light permitting signal is justified (a similar approach, in particular, has already been successfully applied in the street road network of some cities in Austria). At the same time the main purpose of “smart traffic lights” is to monitor traffic flows and select the optimal time points for switching control signals. The greatest difficulties in managing traffic flows are caused by the effect of “wave speed reduction” [20], when, as a result of a vehicle braking at a traffic light, all subsequent drivers inadvertently seek to increase the safe distance, contributing to the formation of traffic congestion. Therefore, it is necessary to study the heterogeneous spatial dynamics of agents and use data on the structure of traffic and pedestrian flows for adaptive traffic light control.

In this article, we propose a new simulation model of heterogeneous traffic flows in a “smart city” with adaptive traffic light behavior control based on fuzzy clustering. Within the framework of such a model, individual decisions on switching traffic light control

signals are based on a fuzzy assessment of the traffic situation, including the evolutionary dynamics of both traffic and pedestrian flows (i.e. with equal priority in relation to cars and pedestrians). At the same time, an important task is solved to maximize the total traffic of the output stream under various scenarios, in particular, for the ITS with spatially homogeneous and periodic flow characteristics.

The scenarios presented in the paper, the corresponding optimal controls, as well as, in general, the proposed universal simulation model with the possibility of further modification of the studied geometry of intersections, as the authors see, can be considered to be an element of an integrated decision-making system in the management of urban services.

### 1. Description of the model

A key fragment of a multi-agent transport system of the Manhattan Lattice type is considered, consisting of four interconnected nodes-intersections that allow arbitrary change of vehicle directions, i.e. movement in a straight line, turns to the left and right, as well as a U-turn and movement in the opposite direction (Fig. 1).

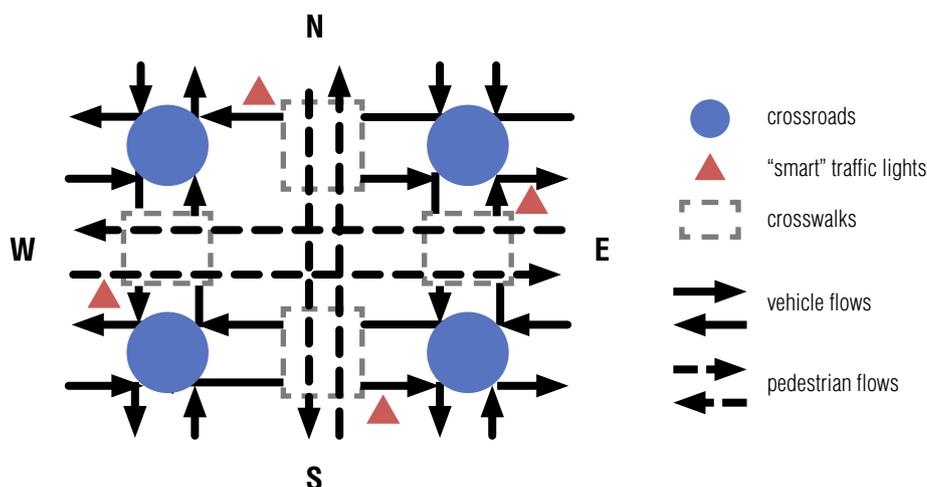


Fig. 1. General scheme of a multi-agent transport system of the Manhattan Lattice type with controlled traffic at pedestrian crossings.

Earlier, in [18, 19], the dynamics of traffic flows in Manhattan Lattice class systems was examined and various ways of their optimization were proposed, mainly based on the management of vehicle routes, i.e. the search and assignment of optimal routes for each agent-vehicle [18], including using genetic algorithms [19]. At the same time, one of the important ways to reduce the load of the ITS is to improve the manoeuvrability of the vehicle, including by choosing the least loaded traffic lanes, the determination of which is implemented using the fuzzy clustering algorithm [21, 22]. The existing methods of improving traffic flow are implemented mainly for unmanned vehicles (UVs), which can be “assigned” the optimal route depending on the current situation. The spatial dynamics of conventional vehicles (CVs) is most influenced by “smart traffic lights” that regulates the movement of traffic and pedestrian flows. In particular, they make it possible to effectively redistribute vehicle flows at intersections and pedestrian crossings, preventing the formation of traffic congestion.

Such traffic jams are formed mainly as a result of the “wave speed reduction.” first studied in [20] and illustrated in Fig. 2. When braking a vehicle, for example, at the stop line of an adjustable pedestrian crossing or in front of the nearest obstacle in the form of another vehicle (Fig. 2), the car following it, as a rule, will brake harder in order to maintain a safe distance by increasing the radius of his personal space, due to the psychological characteristics of the driver’s reaction. Further, the effect of an increase in the “safe” distance (“expansion” of personal space) spreads along the chain, reducing the flow rate as you move away from the original source of congestion (traffic lights), up to a complete stop.

To model the spatial dynamics of agents within the ITS (vehicles and pedestrians), systems of finite-difference equations with a variable structure can be used [21, 23]. This allows one to consider various scenarios of interaction of vehicles with each other and with the external environment (such as V2V, V2P, V2I, etc.) and the influence of the radius of each agent’s personal space.

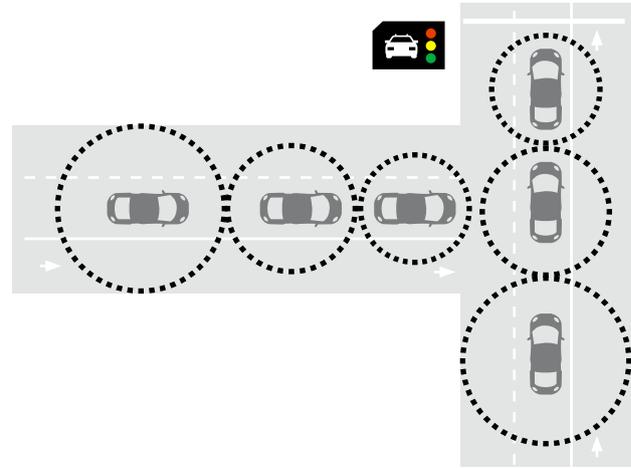


Fig. 2. Illustration of the effect of “wave speed reduction” in a road network with an adjustable pedestrian crossing.

Here is a brief formal description of the developed simulation model of vehicle movement, considering the influence of “smart traffic lights” regulating traffic and pedestrian traffic within the ITS.

Here,

$T = \{t_0, t_1, \dots, |T|\}$  is the set of time moments (in minutes),  $|T|$  is the total number of time moments;  $t_0 \in T$ ,  $t_{|T|} \in T$  are initial and final moments of time;

$L = \{l_0, l_1, \dots, l_{|L|}\}$  is the set of indices of “smart traffic lights”, where  $|L|$  is the total number of “smart traffic lights”;

$s_l(t_{k-1}) \in \{1, 2, 3\}$ ,  $l \in L$  are the states of the phase of the  $l$ -th “smart traffic light” at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ ):  $s_l(t_{k-1}) = 1$  is the prohibiting (for agents-vehicle) traffic light signal (“red”),  $s_l(t_{k-1}) = 2$  is the warning signal of the traffic light (“yellow”),  $s_l(t_{k-1}) = 3$  is the permitting (for agents-vehicle) traffic light signal (“green”);

$\{\tau_{l1}, \tau_{l2}, \tau_{l3}\} \in T$ ,  $l \in L$  is the duration of the phases of the  $l$ -th “smart traffic light” (in seconds) (*control parameter of the model*);

$\tilde{\tau}_l$  is the minimum required (to ensure safe traffic) duration of the main phase (“red” or “green”) for “smart traffic lights” (in seconds) (*control parameter of the model*);

$\{P_l(t_{k-1}), V_l(t_{k-1})\}$  is the total number of pedestrians and vehicles, respectively, located in the monitoring zones of the  $l$ -th “smart traffic light” at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );

$\{\tilde{V}_l(t_{k-1}), \tilde{D}_l(t_{k-1})\}$  is the total number of vehicles in clusters and the average inter-cluster distance calculated using the fuzzy clustering algorithm for agents located in the monitoring zones of the  $l$ -th smart traffic light at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );

$\delta_l, l \in L$  is the threshold ratio between the number of pedestrians at the crossing regulated by the  $l$ -th smart traffic light and the total number of agent-vehicles planning to move this crossing (in any direction), at which it is necessary to turn on the traffic light permitting signal (control parameter of the model);

$\alpha_l, l \in L$  is the coefficient of significance of the average inter-cluster distance (for vehicles), estimated using the fuzzy clustering algorithm, when controlling traffic flows regulated by the  $l$ -th smart traffic light (control parameter of the model);

The phase status of the  $l$ -th “smart traffic light” ( $l \in L$ ) at the moment  $t_k$  ( $t_k \in T$ ) is set up according to the following rules:

$$s_l(t_k) = \begin{cases} 1, & \text{if I, II, or III is true,} \\ 2, & \text{if IV is true,} \\ 3, & \text{if V, VI, or VII is true,} \end{cases} \quad (1)$$

under conditions:

I. ( $t_k \leq t_{k-1} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ ) or ( $t_k > t_{k-1} + \tau_{l2} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ ) with the **first method** of controlling the duration of phases, based on the **collective impact on traffic lights**;

II. ( $(t_k \leq t_{k-1} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ ) or ( $t_k > t_{k-1} + \tau_{l2} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ )) or

$$\left( \frac{P_l(t_{k-1})}{V_l(t_{k-1})} > \delta_l \text{ and } s_l(t_{k-1}) = 3 \text{ and } t_k > t_{k-1} + \tilde{\tau}_l \right)$$

with the **second method** of controlling the duration of phases based on local (“weakly adaptive”) control of traffic light switching, considering the **prioritization of pedestrian traffic**;

III. ( $(t_k \leq t_{k-1} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ ) or

( $t_k > t_{k-1} + \tau_{l2} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ )) or

$$\left( \frac{P_l(t_{k-1})}{\tilde{V}_l(t_{k-1})(\tilde{D}_l(t_{k-1}))^{-\alpha_l}} > \delta_l \text{ and } s_l(t_{k-1}) = 3 \text{ and } t_k > t_{k-1} + \tilde{\tau}_l \right)$$

with the **third (adaptive) method** of controlling the duration of phases based on the fuzzy clustering algorithm, considering the **prioritization of pedestrian traffic**;

IV. ( $(t_k > t_{k-1} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ ) or ( $t_k > t_{k-1} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ )) or ( $t_k \leq t_{k-1} + \tau_{l2}$  and  $s_l(t_{k-1}) = 2$ ), which means that one of the main traffic lights (“red” or “green”) has expired or continues to operate, the previously included warning (“yellow”) signal;

V. ( $t_k \leq t_{k-1} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ ) or ( $t_k > t_{k-1} + \tau_{l2} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ ) with the **first method** of controlling the duration of phases, based on the **collective impact on traffic lights**;

VI. ( $(t_k \leq t_{k-1} + \tau_{l3}$  and  $s_l(t_{k-1}) = 3$ ) or

( $t_k > t_{k-1} + \tau_{l2} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ )) or

$$\left( \frac{P_l(t_{k-1})}{V_l(t_{k-1})} < \delta_l \text{ and } s_l(t_{k-1}) = 1 \text{ and } t_k > t_{k-1} + \tilde{\tau}_l \right)$$

with the **second method** of controlling the duration of phases based on local (“weakly adaptive”) control of traffic light switching, considering the **prioritization of traffic flow traffic**;

VII. ( $(t_k \leq t_{k-1} + \tau_{l1}$  and  $s_l(t_{k-1}) = 3$ ) or

( $t_k > t_{k-1} + \tau_{l2} + \tau_{l1}$  and  $s_l(t_{k-1}) = 1$ )) or

$$\left( \frac{P_l(t_{k-1})}{\tilde{V}_l(t_{k-1})(\tilde{D}_l(t_{k-1}))^{-\alpha_l}} < \delta_l \text{ and } s_l(t_{k-1}) = 1 \text{ and } t_k > t_{k-1} + \tilde{\tau}_l \right)$$

with the **third (adaptive) method** of controlling the duration of phases, based on the **fuzzy clustering algorithm**, considering the **prioritization of the transport stream traffic**.

The total number of vehicles and pedestrians located in the monitoring zones of the  $l$ -th “smart traffic light” ( $l \in L$ ) at the moment  $t_k$  ( $t_k \in T$ ) calculated with the

second method of local (“weakly adaptive”) switching control is equal to

$$V_l(t_k) = \sum_{i=1}^{|I|} v_{il}(t_k), P_l(t_k) = \sum_{\tilde{i}=1}^{|\tilde{I}|} p_{\tilde{i}l}(t_k), \quad (2)$$

where

$$v_{il}(t_k) = \begin{cases} 1, & \text{if } d_{il}(t_k) \leq R_{l1}, \\ 0, & \text{if } d_{il}(t_k) > R_{l1}, \end{cases} \quad (3)$$

$$p_{\tilde{i}l}(t_k) = \begin{cases} 1, & \text{if } \tilde{d}_{\tilde{i}l}(t_k) \leq R_{l1}, \\ 0, & \text{if } \tilde{d}_{\tilde{i}l}(t_k) > R_{l2}, \end{cases}$$

where

$I = \{i_1, i_2, \dots, i_{|I|}\}$  is the set of indices of agent-vehicles, where  $|I|$  is the total number of vehicles;

$\tilde{I} = \{\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_{|\tilde{I}|}\}$  is the set of indices of agent-pedestrians, where  $|\tilde{I}|$  is the total number of pedestrians;

$\{R_{l1}, R_{l2}\}$ ,  $l \in L$  are the radiuses of traffic monitoring zones for road and pedestrian traffic, respectively, for the  $l$ -th “smart traffic light” (*control parameter of the model*);

$\{d_{il}(t_k), \tilde{d}_{\tilde{i}l}(t_k)\}$ ,  $i \in I, \tilde{i} \in \tilde{I}, l \in L$  is the distance from the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian to the  $l$ -th “smart traffic light” at the moment  $t_k$  ( $t_k \in T$ ).

The total number of vehicles in clusters and the average inter-cluster distance for traffic flows located in the monitoring zones of the  $l$ -th “smart traffic light” ( $l \in L$ ) at the moment  $t_k$  ( $t_k \in T$ ) calculated with the third method of adaptive switching control using the fuzzy clustering algorithm are equal

$$\tilde{V}_l(t_k) = \sum_{i=1}^{|I|} \sum_{c_l=1}^{|C_l|} \tilde{v}_{ic_l}(t_k), \tilde{D}_l(t_k) = \frac{1}{|C_l|} \sum_{c_l=1}^{|C_l|} \sum_{\tilde{c}_l=1}^{|C_l|} \hat{d}_{c_l\tilde{c}_l}(t_k), \quad (4)$$

where

$C_l = \{c_{l1}, c_{l2}, \dots, c_{l|C_l|}\}$ ,  $l \in L$  is the set of cluster indices determined for the analysis of the traffic situation in the location area of the  $l$ -th “smart traffic light” using a fuzzy clustering algorithm, where  $|C_l|$  is the total number of clusters (*control parameter of the model*);

$\tilde{v}_{ic_l}(t_k)$ ,  $i \in I, \tilde{i} \in \tilde{I}, l \in L$  is the total number of vehicles belonging to the  $c_l$ -th cluster at the moment  $t_k$  ( $t_k \in T$ );

$\hat{d}_{c_l\tilde{c}_l}(t_k)$ ,  $c_l, \tilde{c}_l \in C_l, c_l \neq \tilde{c}_l, l \in L$  are pairwise distances between the centers of clusters belonging to the  $l$ -th “smart traffic light” at the moment  $t_k$  ( $t_k \in T$ ).

The spatial dynamics of vehicle agents and pedestrians can be modelled using systems of finite-difference equations with the variable structure, considering the regulatory impact of “smart traffic lights.”

Here,

$\{x_{ii}(t_k), y_{ii}(t_k)\}$ ,  $\{\tilde{x}_{\tilde{i}\tilde{i}}(t_k), \tilde{y}_{\tilde{i}\tilde{i}}(t_k)\}$ ,  $i \in I, \tilde{i} \in \tilde{I}, l \in L$  are coordinates of the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian located in the monitoring zone of the  $l$ -th “smart traffic light” at the moment  $t_k$  ( $t_k \in T$ );

$\{v_i(t_{k-1}), \tilde{v}_{\tilde{i}}(t_{k-1})\}$ ,  $i \in I, \tilde{i} \in \tilde{I}$  is the preferred speed of the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );

$\{r_i(t_{k-1}), \tilde{r}_{\tilde{i}}(t_{k-1})\}$ ,  $i \in I, \tilde{i} \in \tilde{I}, l \in L$  are the radius of personal spaces of the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian, the values of which depend on the density of the transport (pedestrian) flow consisting of agents that reduce their speed and are located in the direction of travel (see Fig. 2) at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );

$\{m_{ib}(t_k), \tilde{m}_{\tilde{i}b}(t_k)\}$ ,  $i \in I, \tilde{i} \in \tilde{I}, b \in I \cup \tilde{I}$  is the distance from the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian to the nearest  $b$ -th agent-obstacle at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );

$\{w_i(t_{k-1}), \tilde{w}_{\tilde{i}}(t_{k-1})\}$ ,  $\{q_i(t_{k-1}), \tilde{q}_{\tilde{i}}(t_{k-1})\} \in \{-1, 0, 1\}$ ,  $i \in I$ , are parameters that determine the direction of movement of the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ ):

$w_i(t_{k-1}), \tilde{w}_{\tilde{i}}(t_{k-1}) = -1$  when moving in the direction of the **E-W** (see Fig. 1),

$w_i(t_{k-1}), \tilde{w}_{\tilde{i}}(t_{k-1}) = 0$  when moving in the direction of the **N-S** or **S-N**,

$w_i(t_{k-1}), \tilde{w}_{\tilde{i}}(t_{k-1}) = 1$  when moving in the direction of the **W-E**,

$q_i(t_{k-1}), \tilde{q}_{\tilde{i}}(t_{k-1}) = -1$  when moving in the direction of the **S-N**,

$q_i(t_{k-1}), \tilde{q}_i(t_{k-1}) = 0$  when moving in the direction of the **W-E** or **E-W**,

$q_i(t_{k-1}), \tilde{q}_i(t_{k-1}) = 1$  when moving in the direction of the **S-N**;

$\lambda$  is the coefficient that specifies the ratio of the scales of real and model time.

The spatial dynamics of the  $i$ -th agent-vehicle ( $i \in I$ ) and the  $i$ -th agent-pedestrian ( $i \in I$ ), located in the monitoring zone of the  $l$ -th smart traffic light ( $l \in L$ ) at the moment  $t_k$  ( $t_k \in T$ ) without taking into account internal manoeuvring (associated with overtaking, lane changes, etc.) is given by the following system of finite difference equations with a variable structure:

$$x_{il}(t_k) = \begin{cases} x_{il}(t_{k-1}) + w_i(t_{k-1})\lambda v_i(t_{k-1}), & \text{if VIII is true,} \\ x_{il}(t_{k-1}), & \text{if IX is true,} \end{cases} \quad (5)$$

$$y_{il}(t_k) = \begin{cases} y_{il}(t_{k-1}) + q_i(t_{k-1})\lambda v_i(t_{k-1}), & \text{if VIII is true,} \\ y_{il}(t_{k-1}), & \text{if IX is true,} \end{cases} \quad (6)$$

$$\tilde{x}_{\tilde{i}l}(t_k) = \begin{cases} \tilde{x}_{\tilde{i}l}(t_{k-1}), & \text{if X is true,} \\ \tilde{x}_{\tilde{i}l}(t_{k-1}) + \tilde{w}_{\tilde{i}}(t_{k-1})\lambda \tilde{v}_{\tilde{i}}(t_{k-1}), & \text{if XI is true,} \end{cases} \quad (7)$$

$$\tilde{y}_{\tilde{i}l}(t_k) = \begin{cases} \tilde{y}_{\tilde{i}l}(t_{k-1}), & \text{if X is true,} \\ \tilde{y}_{\tilde{i}l}(t_{k-1}) + \tilde{q}_{\tilde{i}}(t_{k-1})\lambda \tilde{v}_{\tilde{i}}(t_{k-1}), & \text{if XI is true,} \end{cases} \quad (8)$$

$i \in I, \tilde{i} \in \tilde{I}, b \in I \cup \tilde{I}, l \in L,$

where

VIII.  $s_i(t_{k-1}) = 3$  and  $m_{ib}(t_{k-1}) > (r_i(t_{k-1}) + r_b(t_{k-1}))$  for the nearest agent ( $b \in I \cup \tilde{I}$ ), which means that the permissive (for agent-vehicles) traffic light signal (“green”) is in effect and there are no obstacles in the form of other vehicles or pedestrians on the way of the  $i$ -th agent-vehicle ( $i \in I$ );

IX.  $s_i(t_{k-1}) = 1$  and  $m_{ib}(t_{k-1}) \leq (r_i(t_{k-1}) + r_b(t_{k-1}))$  for the nearest agent ( $b \in I \cup \tilde{I}$ ), which means that a prohibitor (for agent-vehicles) traffic light signal (“red”) is in effect, or there is an obstacle in the form of another vehicle or a pedestrian on the way of the  $i$ -th agent-vehicle ( $i \in I$ );

X.  $s_i(t_{k-1}) = 1$  and  $\tilde{m}_{\tilde{i}b}(t_{k-1}) > (\tilde{r}_{\tilde{i}}(t_{k-1}) + r_b(t_{k-1}))$  for the nearest agent ( $b \in I \cup \tilde{I}$ ), which means that a prohibitor (for agent-vehicles) traffic light signal (“red”) is in effect and there are no obstacles in the form of other pedestrians or vehicles on the way of the  $\tilde{i}$ -th agent-pedestrian ( $\tilde{i} \in \tilde{I}$ );

XI.  $s_i(t_{k-1}) = 1$  or  $\tilde{m}_{\tilde{i}b}(t_{k-1}) \leq (\tilde{r}_{\tilde{i}}(t_{k-1}) + r_b(t_{k-1}))$  for the nearest agent ( $b \in I \cup \tilde{I}$ ), which means that the permissive (for agent-vehicles) traffic light signal (“green”) is in effect, or there is an obstacle in the form of another pedestrian or vehicle on the way of the  $\tilde{i}$ -th agent-pedestrian ( $\tilde{i} \in \tilde{I}$ ).

The total traffic of the output stream that should be maximized is equal to

$$N = \sum_{t_k=0}^{|T|} \left( \sum_{i=1}^{|I|} n_i + \sum_{\tilde{i}=1}^{|\tilde{I}|} \tilde{n}_{\tilde{i}} \right), \quad (9)$$

where

$$n_i(t_k) = \begin{cases} 1, & \text{if } \{x_i(t_{k-1}), y_i(t_{k-1})\} \notin \{X, Y\}, \\ 0, & \text{if } \{x_i(t_{k-1}), y_i(t_{k-1})\} \in \{X, Y\}, \end{cases} \quad (10)$$

$$\tilde{n}_{\tilde{i}}(t_k) = \begin{cases} 1, & \text{if } \{\tilde{x}_{\tilde{i}}(t_{k-1}), \tilde{y}_{\tilde{i}}(t_{k-1})\} \notin \{X, Y\}, \\ 0, & \text{if } \{\tilde{x}_{\tilde{i}}(t_{k-1}), \tilde{y}_{\tilde{i}}(t_{k-1})\} \in \{X, Y\}, \end{cases} \quad (11)$$

where

$\{x_i(t_{k-1}), y_i(t_{k-1})\}, \{\tilde{x}_{\tilde{i}}(t_{k-1}), \tilde{y}_{\tilde{i}}(t_{k-1})\}, i \in I, \tilde{i} \in \tilde{I}$  are coordinates of the  $i$ -th agent-vehicle and the  $\tilde{i}$ -th agent-pedestrian within the ITS at the moment  $t_{k-1}$  ( $t_{k-1} \in T$ );  $\{X, Y\}$  is the set of all coordinates of the ITS digital road network.

Then, it is possible to formulate the following optimization problem to be solved considering the chosen method of controlling “smart traffic lights.”

**Problem A.** The need to maximize the total traffic of the output flow by the set of control parameters  $\{\tau_{11}, \tau_{12}, \tau_{13}, \tilde{\tau}_i, \delta_i, R_{11}, R_{12}, |C_i|, \alpha_i\}$ :

$$\max_{\{\tau_{11}, \tau_{12}, \tau_{13}, \tilde{\tau}_i, \delta_i, R_{11}, R_{12}, |C_i|, \alpha_i\}} N \quad (12)$$

s.t.

$$\tau_{11}, \tau_{12}, \tau_{13}, \tilde{\tau}_i \in [\underline{\tau}, \bar{\tau}], \delta_i \in [\underline{\delta}, \bar{\delta}], R_{11} \in [\underline{R}, \bar{R}_1], R_{21} \in [\underline{R}_2, \bar{R}_2], |C_i| \in [\underline{C}, \bar{C}], \alpha_i \in [\underline{\alpha}, \bar{\alpha}],$$

where  $\{\underline{\tau}, \underline{\delta}, \underline{R}_1, \underline{R}_2, \underline{C}, \underline{\alpha}\}$ ,  $\{\overline{\tau}, \overline{\delta}, \overline{R}_1, \overline{R}_2, \overline{C}, \overline{\alpha}\}$  are the lower and upper boundary values of the control parameters of the model.

To solve **Problem A**, the previously proposed genetic optimization algorithm of real coding (RCGA class) is used [20, 25], aggregated by target functionality with the developed simulation model of the transport system implemented in AnyLogic.

## 2. Fuzzy clustering algorithm

To assess the structure of traffic flow and adaptive control of “smart traffic lights”, it is proposed to use the fuzzy clustering algorithm (Fuzzy C-means) [21, 22, 26, 27]. The choice of this algorithm is primarily due to the possibility of considering various characteristics of moving vehicles in the formation of clusters, in particular, density, speed, distance from the traffic light regulating traffic at the transition, etc. The inclusion of such characteristics in cluster analysis makes it possible to achieve maximum “likelihood” when assessing the structure of the traffic flow. Unlike classical algorithms, Fuzzy C-means does not assign an object unambiguously to any cluster, but compares each cluster with the probability of assigning observed objects to it, forming a so-called membership matrix.

The enlarged scheme of the proposed fuzzy clustering algorithm is shown in *Fig. 3*. An important difference between the developed algorithm and those previously known is that its key characteristics (for example, the number of clusters, the radius of the traffic monitoring zone, etc.) are calculated using a genetic optimization algorithm (RCGA class) as part of solving the main problem of maximizing output stream traffic. As a result, the results of fuzzy clustering directly affect the possibilities of finding optimal solutions for the ITS being studied.

*Figure 3* uses the following notation:

- ◆  $z \in [0, 1]$  is the measure of fuzziness;
- ◆  $M(k)$  is the membership matrix at the  $k$ th step of the algorithm,  $k = 1, 2, \dots, |K|$  where  $|K|$  is the maximum number of iterations;
- ◆  $\varepsilon$  is a small parameter that is a criterion for the algorithm stopping.

Thus, the proposed fuzzy clustering algorithm is aggregated by the target functional (the total traffic of the ITS output stream), with the real-coded genetic algorithm (*Fig. 3*). RCGA uses heuristic crossing-over and mutation operators (for example, LX, SBX, SNUM, see [21, 24, 25]) to form new potential solutions with the best characteristics. The Fuzzy C-means algorithm was built into the ITS simulation model implemented in AnyLogic and is executed at each step of the model time, providing an assessment of the structure of the traffic flow located in the monitoring area of each “smart traffic light.”

## 3. Software implementation of the model

The key fragment of the software implementation of the proposed ITS simulation model performed in the AnyLogic environment is shown in *Fig. 4*.

An important feature of the software implementation of the model (*Fig. 4*) is the combined use of discrete-event and agent methods, including those supported in the AnyLogic traffic library [28, 29]. In particular, elements of the **carSource** and **pedSource** types provide generation of new agents and their addition to the corresponding populations of vehicles and pedestrians, elements of the **SelectOutput** type ( $s1, s2$  in *Fig. 4b*) are used to distribute traffic flow along possible routes when the vehicle reaches intersections; **CarMoveTo** and **pedGoTo** elements move vehicle agents and pedestrians to a given goal, according to their predefined characteristics (preferred speed, intensity of arrival, etc.); **carDispose** and **pedSink** ensure the removal of agents from the corresponding populations and the calculation of the output traffic.

## 4. Results of optimization experiments

Optimization experiments were carried out for the ITS with spatially homogeneous and periodic flow characteristics with three methods of controlling “smart traffic lights”:

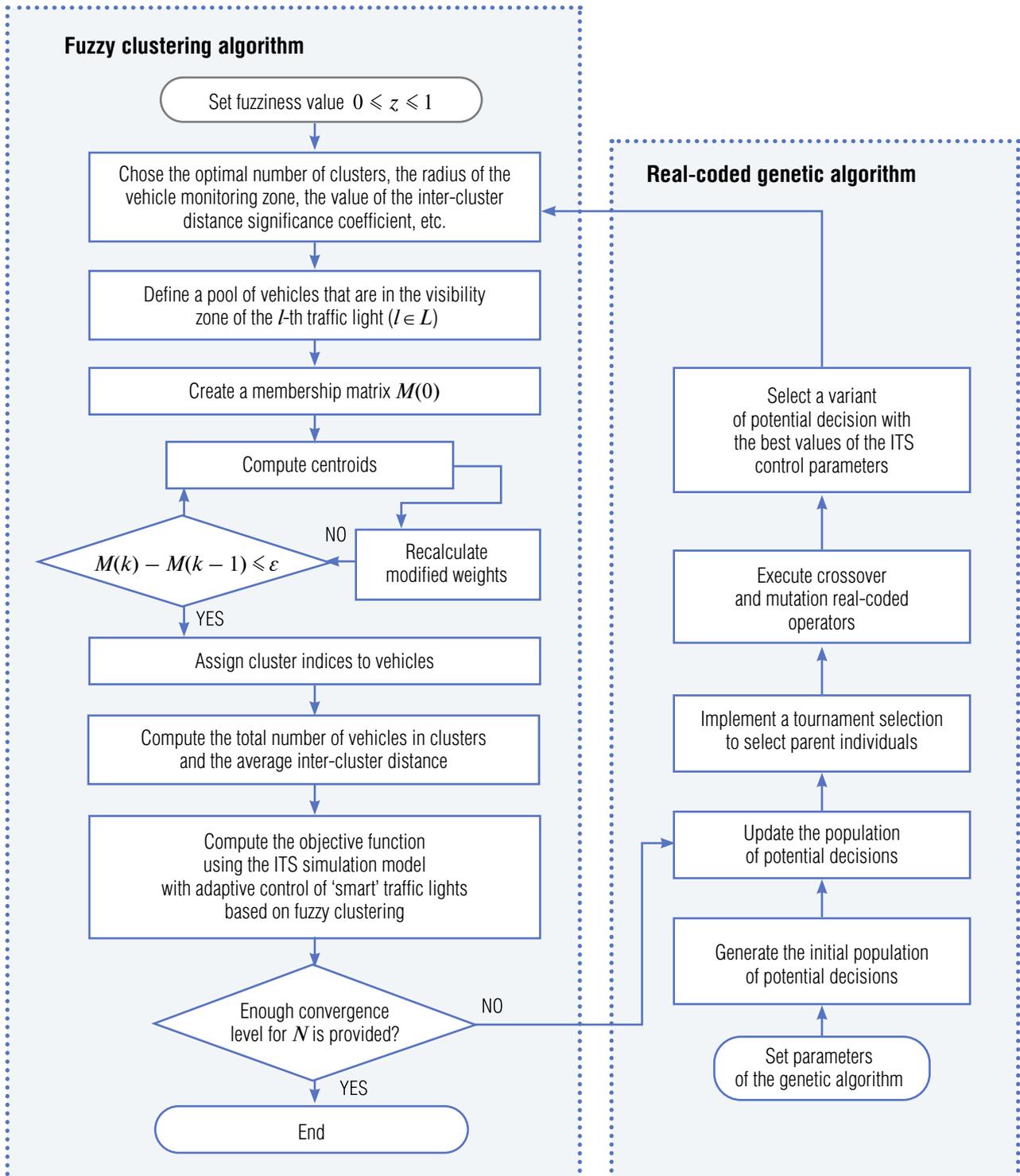


Fig. 3. Fuzzy clustering algorithm for adaptive traffic light control.

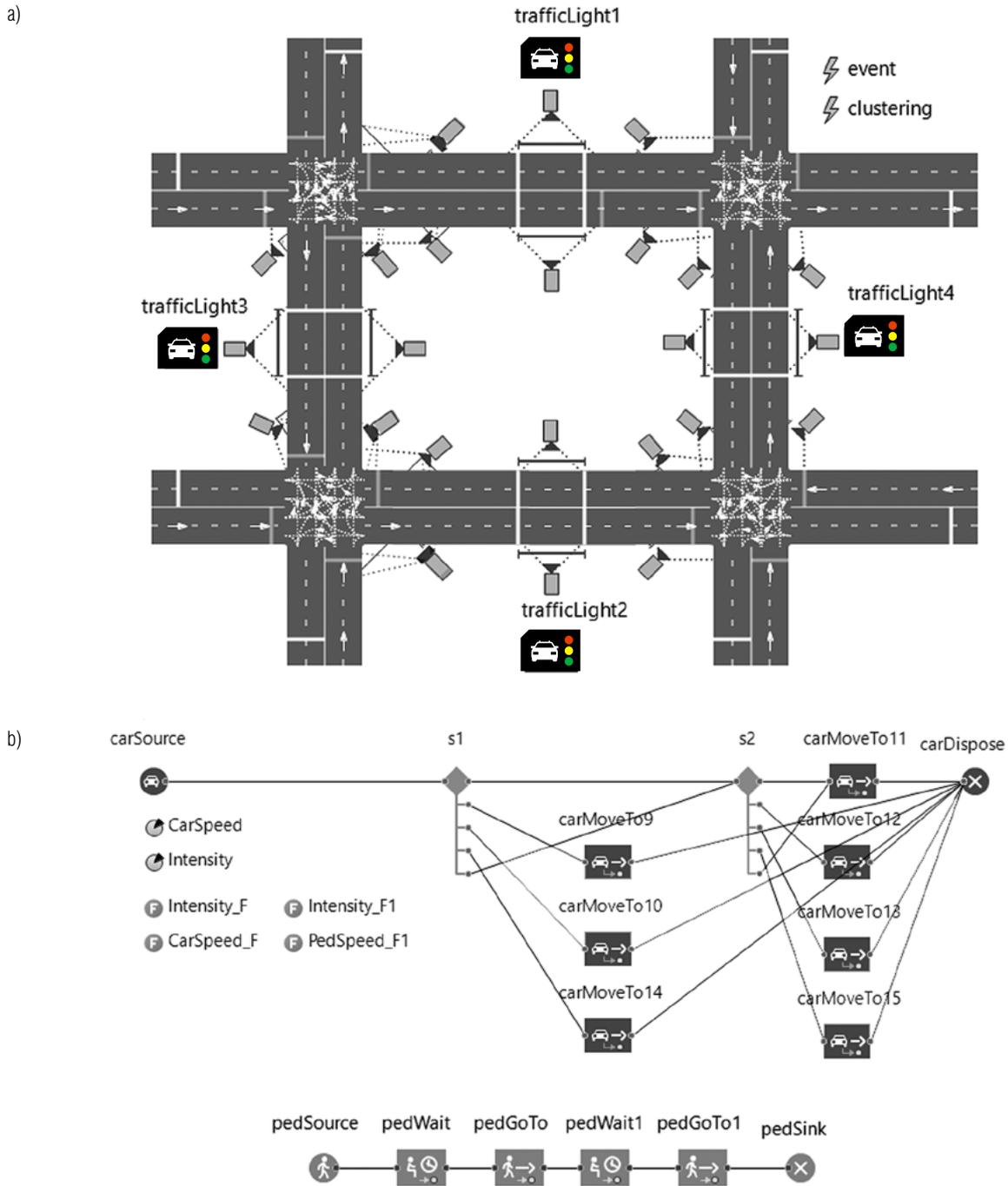


Fig. 4. Software implementation of the ITS simulation model in AnyLogic:  
 a) a diagram of a digital road network with "smart traffic lights"  
 b) a fragment of a discrete-event model of the movement of agents-vehicles and pedestrians along specified routes.

- ◆ by forming the required duration of phases using a genetic optimization algorithm;
- ◆ using local (“weakly adaptive”) switching control.
- ◆ based on the proposed fuzzy clustering algorithm.

For a system with spatially homogeneous flow characteristics, the intensity of arrival of agents and their preferred speeds are constants.

The intensity of the arrival of agents the ITS with periodic characteristics simulating the presence and absence of peak loads is calculated at each moment of time  $t_k (t_k \in T)$ :

$$\theta(t_k) = \begin{cases} \text{truncnormal}(\hat{\theta}, \sigma, \underline{\theta}, \bar{\theta}), & \text{if } t_k \leq \frac{1}{4}|T|, \\ \tilde{\theta}, & \text{if } \frac{1}{4}|T| < t_k < \frac{3}{4}|T|, \\ \text{truncnormal}(\hat{\theta}, \sigma, \underline{\theta}, \bar{\theta}), & \text{if } t_k \geq \frac{3}{4}|T|, \end{cases} \quad (13)$$

where

$\text{truncnormal}(\hat{\theta}, \sigma, \underline{\theta}, \bar{\theta})$  is the random value of the intensity of the arrival of agents, set using a truncated normal distribution with the mean  $\hat{\theta}$ , standard deviation  $\sigma$ , lower and upper boundary values  $\underline{\theta}, \bar{\theta}$ , corresponding to the conditions of extreme traffic;

$\tilde{\theta}$  is the intensity of arrival corresponding to the conditions of normal traffic;

In a similar way, the average speeds of the agents are set. The main model assumptions (initial data) are presented in *Table 1*.

At the first stage, using the Monte Carlo type method [30], numerical experiments were carried out to assess the sensitivity of the target functional (the total traffic of the output stream) with respect to the values of the ITS control parameters with spatially homogeneous and periodic flow characteristics (*Fig. 5*).

It follows from *Fig. 5* that the total traffic of the output stream is sensitive with respect to the values of the ITS control parameters, both with spatially homogeneous and periodic flow characteristics. At the same time, the most likely ranges of values of the total traffic of the output stream are 1800–1900 agents (vehicles and pedestrians).

*Table 1.*

**Initial data of the simulation model**

No.	Model parameters	Values	
		vehicles	pedestrians
1	Length and width of roads, m.	155	
2	Number of intersections	4	
3	Distance between adjacent intersections, m.	65	
4	Number of traffic lanes for each road	2	
5	The width of the dividing strip, m.	2	
6	The number of pedestrian crossings regulated by “smart traffic lights”	4	
7	Simulation period, min.	20	
8	The intensity of the arrival of vehicles and pedestrians to the ITS with spatially homogeneous characteristics at each entrance of the road network (agents per hour)	vehicles	1000
		pedestrians	500
9	Preferred speed of vehicles and pedestrians within the ITS with spatially homogeneous characteristics (km/h for vehicles and m/s for pedestrians)	vehicles	100
		pedestrians	0.75
10	Parameters for calculating the intensity of arrival of vehicles and pedestrians to the ITS with periodic flow characteristics (agents per hour)	$\hat{\theta}$	500
		$\sigma$	100
		$\underline{\theta}$	100
		$\bar{\theta}$	1500
		$\tilde{\theta}$	100
11	Parameters for calculating the preferred speed of vehicles and pedestrians within the ITS with periodic flow characteristics (km/h for vehicles and m/s for pedestrians)	$\hat{\theta}$	45
		$\sigma$	10
		$\underline{\theta}$	20
		$\bar{\theta}$	60
		$\tilde{\theta}$	100

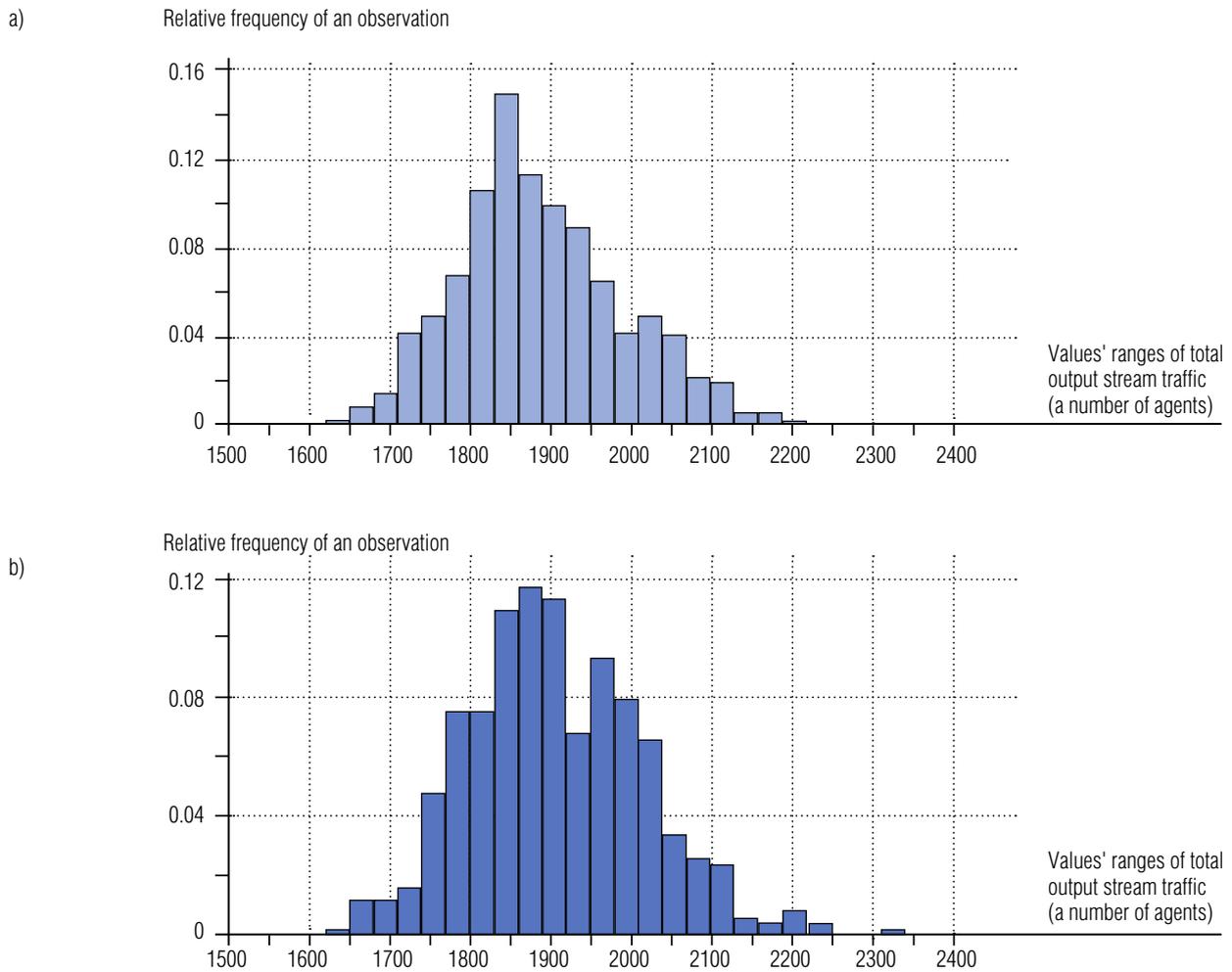


Fig. 5. Estimation of sensitivity (probability density) of the total output stream traffic for the ITS: a) with spatially homogeneous and b) periodic flow characteristics.

Figure 6 shows the dynamics of convergence of the objective function obtained using the developed simulation model aggregated by the target functional with a genetic algorithm (GA).

The maximum possible values of the total output stream for the ITS with periodic flow characteristics are, on average, less than for the ITS with spatially homogeneous characteristics (Fig. 5). The obtained suboptimal values of the control parameters

of the model corresponding to the scenarios of the ITS implementation in an enlarged form discussed above are presented in Table 2.

It follows from Fig. 6 and Table 2 that the most promising way to control “smart traffic lights” is adaptive switching control based on fuzzy clustering. The proposed approach demonstrates its effectiveness even for the ITS with periodic flow characteristics, providing the best final value of the objective function.

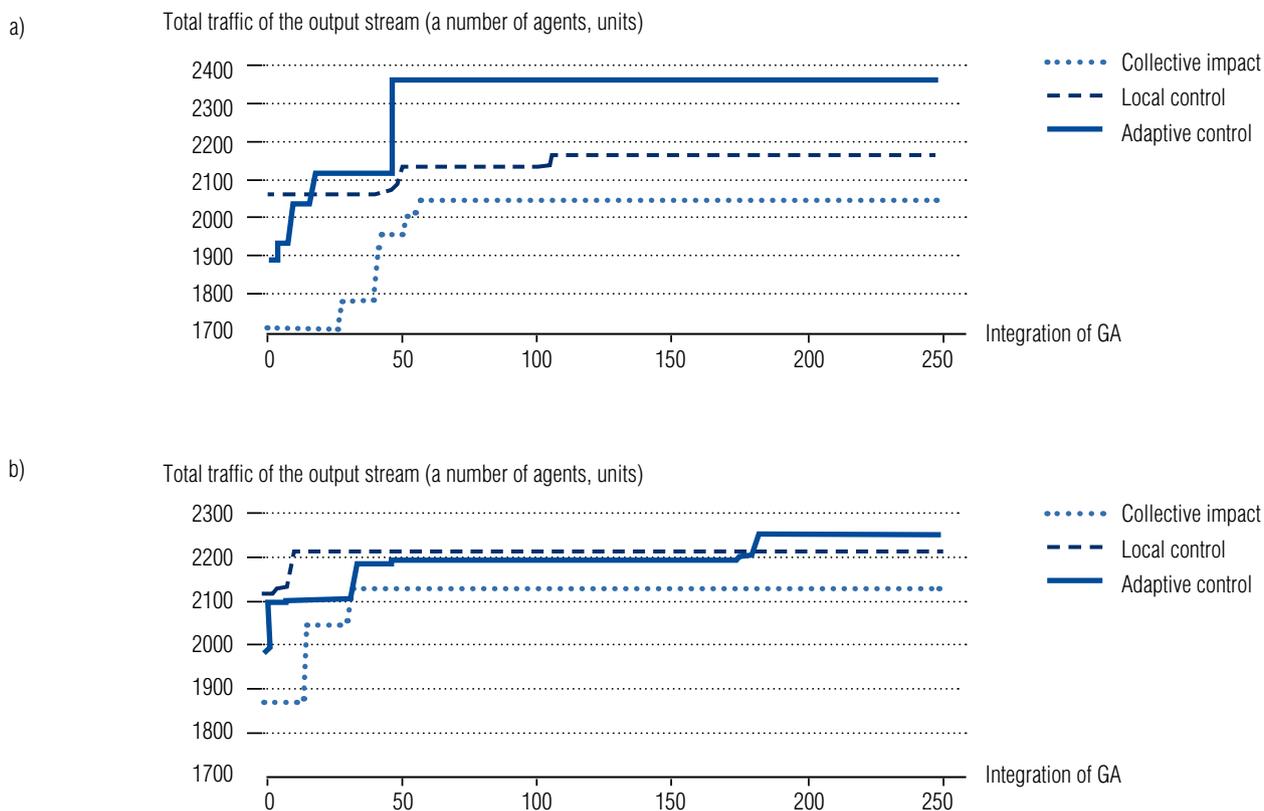


Fig. 6. Dynamics of convergence of the objective function (total traffic of the output stream) for the ITS: a) with spatially homogeneous and b) periodic flow characteristics.

### Conclusion

This article presents a new simulation model of the intelligent transport system (ITS) of a “smart city” with adaptive traffic light control. We propose a model of the movement of a vehicle ensemble using systems of finite-difference equations with a variable structure, considering the regulatory effect of “smart traffic lights.” To assess the structure of traffic flow and adaptive control of “smart traffic lights.” a fuzzy clustering algorithm is proposed, the key characteristics of which are calculated using a genetic optimization algorithm (RCGA class) as part of solving the main task of maximizing output traffic. With the help of the devel-

oped simulation model, the possibilities of rational management of “smart traffic lights” are investigated, in particular, for ITS with spatially homogeneous and periodic characteristics. As a result, a model example demonstrates the greater efficiency of adaptive switching control based on fuzzy clustering.

Further research will be aimed at designing the large-scale agent-based model of the ITS “smart city” using the FLAME GPU. ■

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Table 2.

**The obtained suboptimal values  
of the control parameters of the model**

			<b>A system with spatially homogeneous flow characteristics</b>	<b>A system with periodic flow characteristics</b>
<b>Collective impact</b>				
<b>Total traffic, agents</b>			<b>2392</b>	<b>2125</b>
Duration of traffic light phases, sec.	first and second traffic lights	red	11.327	22.675
		green	25.816	18.061
	third and fourth traffic lights	red	10.136	18.768
		green	11.933	103.071
	all traffic lights	yellow	1.098	1.874
<b>Local management</b>				
<b>Total traffic, agents</b>			<b>2166</b>	<b>2209</b>
Minimum required duration of the main phase, min.			1.839	4.918
Radius of the traffic monitoring zone, m.			31.59	12.25
Radius of the pedestrian traffic monitoring zone, m.			19.13	2.56
Threshold ratio between the number of pedestrians at the crossing and the total number of vehicles			88.5	241.7
<b>Adaptive management based on fuzzy clustering</b>				
<b>Total traffic, agents</b>			<b>2357</b>	<b>2246</b>
Minimum required duration of the main phase, min.			1.514	3.555
Radius of the traffic monitoring zone, m.			26.11	17.89
Radius of the pedestrian traffic monitoring zone, m.			20.21	25.01
Threshold ratio between the number of pedestrians at the crossing and the total number of vehicles (adjusted for inter-cluster distance)			162.5	255.1
Coefficient of significance of the average inter-cluster distance			0.602	1
Number of clusters			3	3

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